

Using STACK on flipped math courses

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International Meeting of the STACK Community 2021

Joint work with

Technology supported mathematics education research group

... any many
other teachers,
teaching
assistants,
collaborators
- and students!



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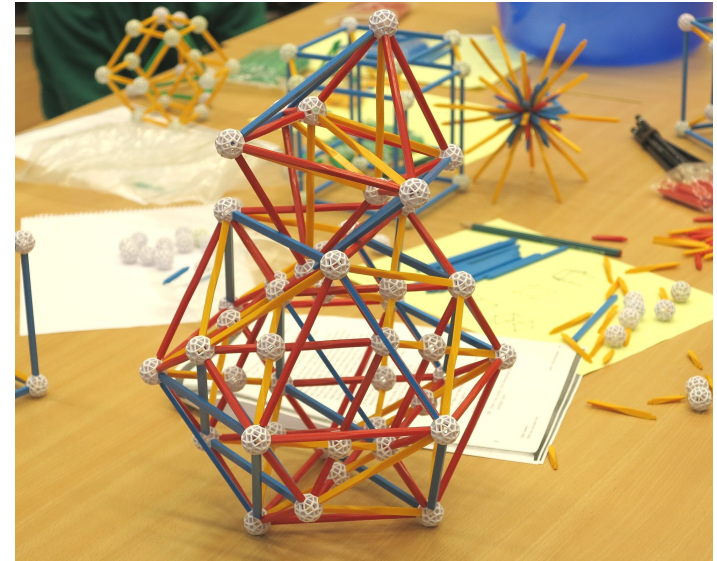


Vikke Vuorenpää

Engineering mathematics at Tampere University

Four mandatory courses, 5 credits each:

- Fundamentals of engineering mathematics
- **Vectors and matrices**
- Differential and integral calculus
- Introduction to probability and statistical inference



"Flipped" implementations for about 300 students yearly, another 300-400 students study the courses more traditionally.

Vectors and matrices

Vectors and matrices

- Students from three Tampere University campuses & FiTech Network University
 - FiTech is collaborative project of seven Finnish universities of technology that offer university courses free of charge to degree students and adult learners (graduates and company representatives)
- Teaching based on flipped learning ideology: online learning platform, written material, lots of small problems to work on, videos, discussion forum, problem sessions, small group meetings with teacher
- Weight of continuous assessment 70-85%, final exam 15-30%
 - Students collect points from online problems, written solutions to homework, self- and peer-assessments, learning portfolio / group work, and final exam

Vectors and matrices: self-study phase

Students study each week a new topic with provided online material:

- Course notes with theory, examples and problems
- STACK problem set, linked with theory
- Short videos

Pohdi 3.1.1

Ruska ja Tuisku ovat lähdössä ruokaostoksille ja vertailevat hintoja kahdessa lähikaupassaan. Alla olevassa taulukossa on heidän kauppalistansa sekä ruokatavaroiden hinnat eri kaupoissa:

	maitoja	sämpylöitä	jogurtteja		Y-kauppa	T-valinta
Ruska	6	4	4	maito	1,40 e	1,30 e
Tuisku	6	2	3	sämpylä	1,10 e	1,15 e
				jogurtti	0,50 e	0,60 e

1. Kuinka monta jogurttia Tuisku aikoo ostaa?
2. Mitä kaikkea Ruska on ostamassa ja kuinka paljon?
3. Mitkä ovat tuotteiden hinnat T-valinnassa?

Kuten Ruskan ja Tuiskun tapauksesta nähdään toisinaan asiat on kätevää kirjoittaa muistiin taulukkoon. Matematiikassa lukutaulukkoja kutsutaan matriiseiksi. Esimerkiksi Ruskan ja Tuiskun kauppalistan voi kirjoittaa matriisinä

$$A = \begin{bmatrix} 6 & 4 & 4 \\ 6 & 2 & 3 \end{bmatrix}$$

ja ruokatavaroiden hinnat matriisiksi

$$B = \begin{bmatrix} 1,40 & 1,30 \\ 1,10 & 1,15 \\ 0,50 & 0,60 \end{bmatrix}.$$

Pisteet 0 / 1 Palautukseni 0 / 5 Määräaika tiistai 1.6.2021 22:00

Tehtävä 1

Kysymys 1

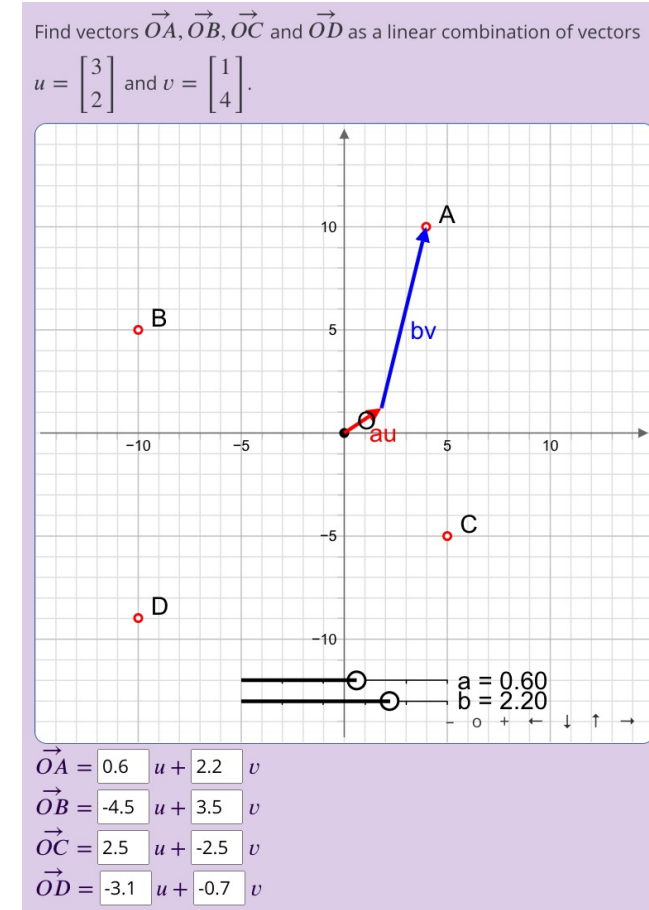
1 piste Valitse paikkansa pitävät väittämät. Oletetaan, että $A, B \in \mathbb{R}^{m \times n}$ ja että $\mathbf{b} \in \mathbb{R}^m \setminus \{\mathbf{0}\}$.

- ☐ Jos \mathbf{s} on matriisiyhtälön $A\mathbf{x} = \mathbf{0}$ ratkaisu, niin myös $2\mathbf{s}$ on ratkaisu.
- ☐ Jos \mathbf{s} on matriisiyhtälön $A\mathbf{x} = \mathbf{b}$ ratkaisu, niin myös $2\mathbf{s}$ on ratkaisu.
- ☐ Jos A on matriisi ja $A\mathbf{x} = \mathbf{0}$ kaikilla vektoreilla \mathbf{x} , niin $A = \mathbf{0}$ on nollamatriisi.
- ☐ Jos $AB = \mathbf{0}$, niin $A = \mathbf{0}$ tai $B = \mathbf{0}$.
- ☐ Jos A ja B ovat neliömatriiseja, niin $(A + B)^2 = A^2 + 2AB + B^2$.

Lähetä

Vectors and matrices: self-study phase

- Each learning week, the students had about 15 STACK problems to solve
- The idea of STACK problems was not only to test the students' knowledge but also to support their learning process and deepen their understanding



Vectors and matrices: self-study phase

Find two vectors \mathbf{u} and \mathbf{v} that span the plane $-4x + 5y + 6z = 0$

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 2/3 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ -(5/6) \end{bmatrix}$$

Do vectors \mathbf{u} and \mathbf{v} span the space \mathbb{R}^2 ? No

Is it possible to write every vector in \mathbb{R}^2 as a linear combination of vectors $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$? Yes

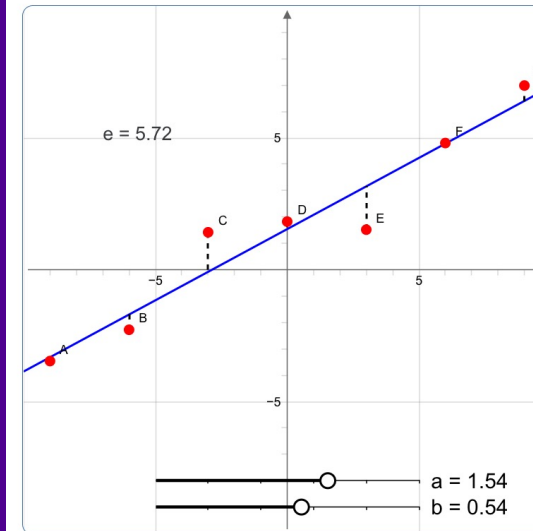
Is it possible to write every vector in \mathbb{R}^2 as a linear combination of vectors $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$? No

- Wide variety of different STACK questions
 - MCQ
 - JSXGraph questions
 - Questions where students use MATLAB to find solution
 - Etc.

Fit the line to the given data using ordinary least squares.

$$X = \begin{bmatrix} -9 & -3.464 \\ -6 & -2.277 \\ -3 & 1.419 \\ 0 & 1.825 \\ 3 & 1.519 \\ 6 & 4.803 \\ 9 & 6.983 \end{bmatrix}$$

Return the parameters a and b of the line $y = bx + a$.



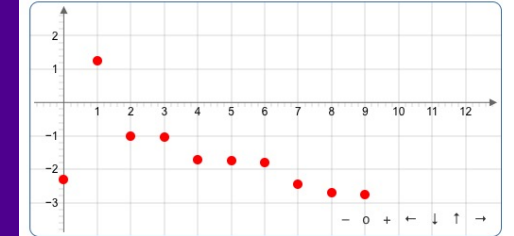
$$y = 0.54x + 1.54$$

The error term is now $e = 5.72$

Fit a linear polynomial to the given set of points. Points are plotted to the figure below. The coordinates of the points are

$x = [0; 1; 2; 3; 4; 5; 6; 7; 8; 9]$ and

$y = [-(23/10); 5/4; -1; -(21/20); -(17/10); -(7/4); -(9/5); -(49/20); -(27/10); -(11/4)]$.



a) Solve the coefficients of linear polynomial $a * x + b$ using MATLAB. Find the coefficients using command `A\b` and try also command `polyfit`. You will get more information about `polyfit` writing `help polyfit` or `doc polyfit` to the MATLAB Command Window.

Answer:

$$\begin{bmatrix} -0.25 \\ -0.5 \end{bmatrix}$$

Your last answer was interpreted as follows:

$$\begin{bmatrix} -0.25 & -0.5 \end{bmatrix}$$

✓ Correct answer, well done.

b) Plot the fitted line to the same figure with the given points. How many of the points are at the line? Hint: `linspace`, `polyval`, `plot`, `hold on`, `hold off`

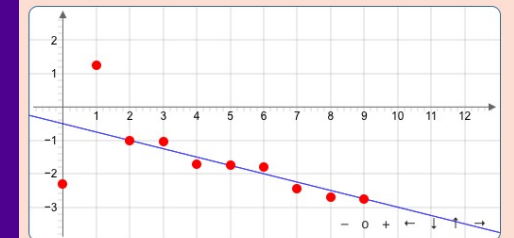
Answer: 3

Your last answer was interpreted as follows:

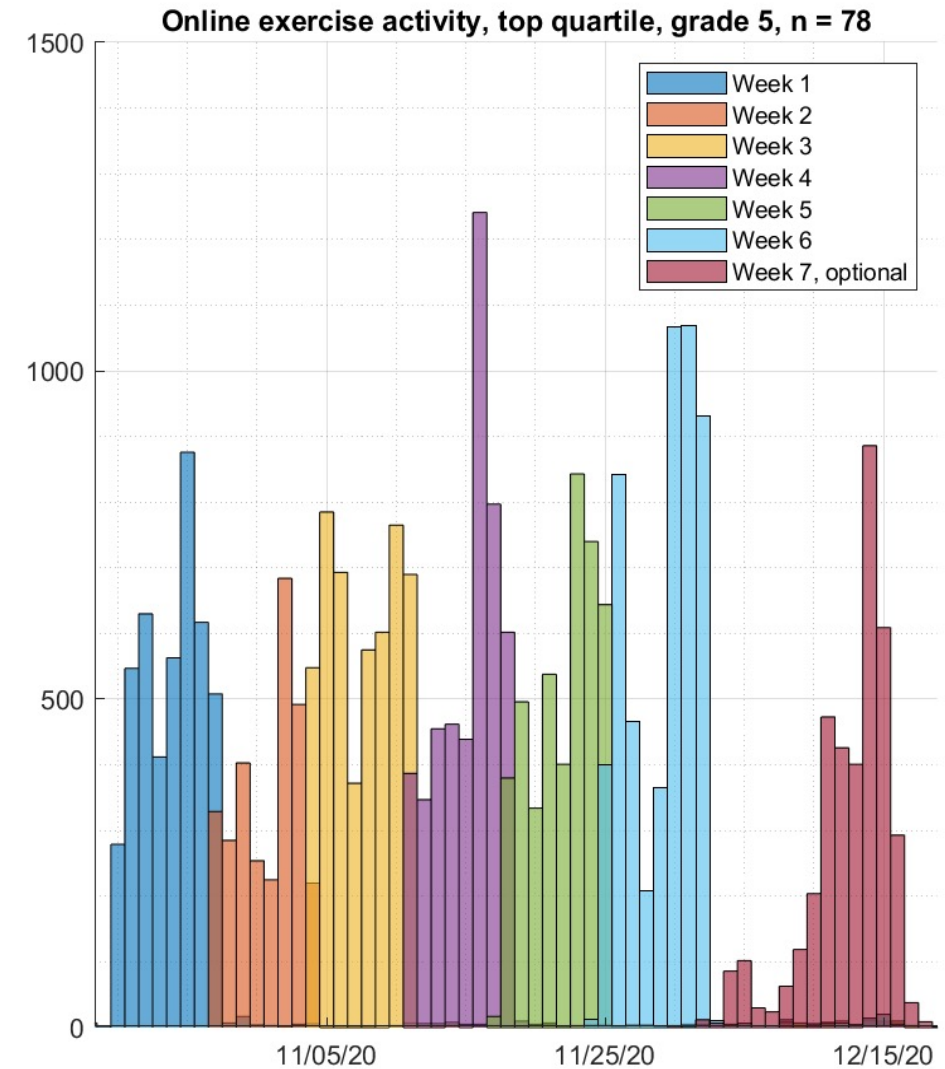
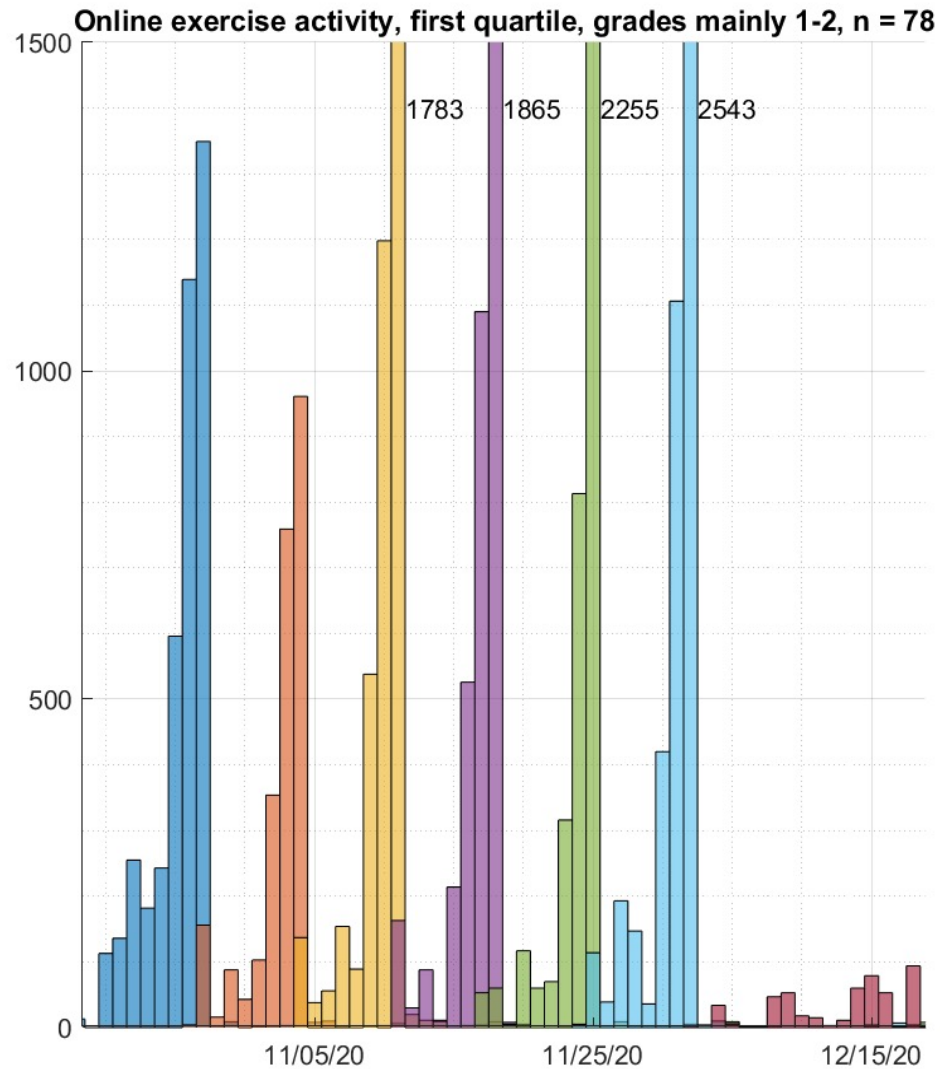
3

✓ Correct answer, well done.

✓ Correct answer, well done.



Vectors and matrices: Online exercise activity



Vectors and matrices: group study phase

Problem sessions

- Several small groups and teaching assistant(s)
- Students discuss solutions to pre-problems and work together on homework problems

Prime time meetings

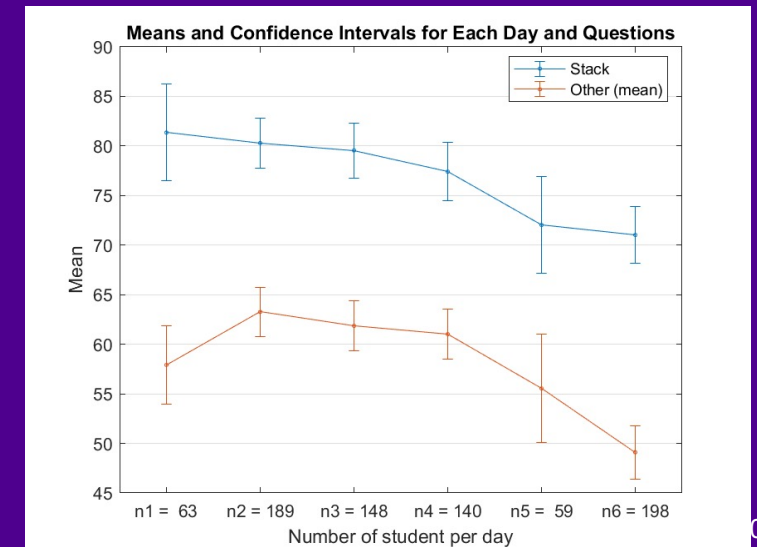
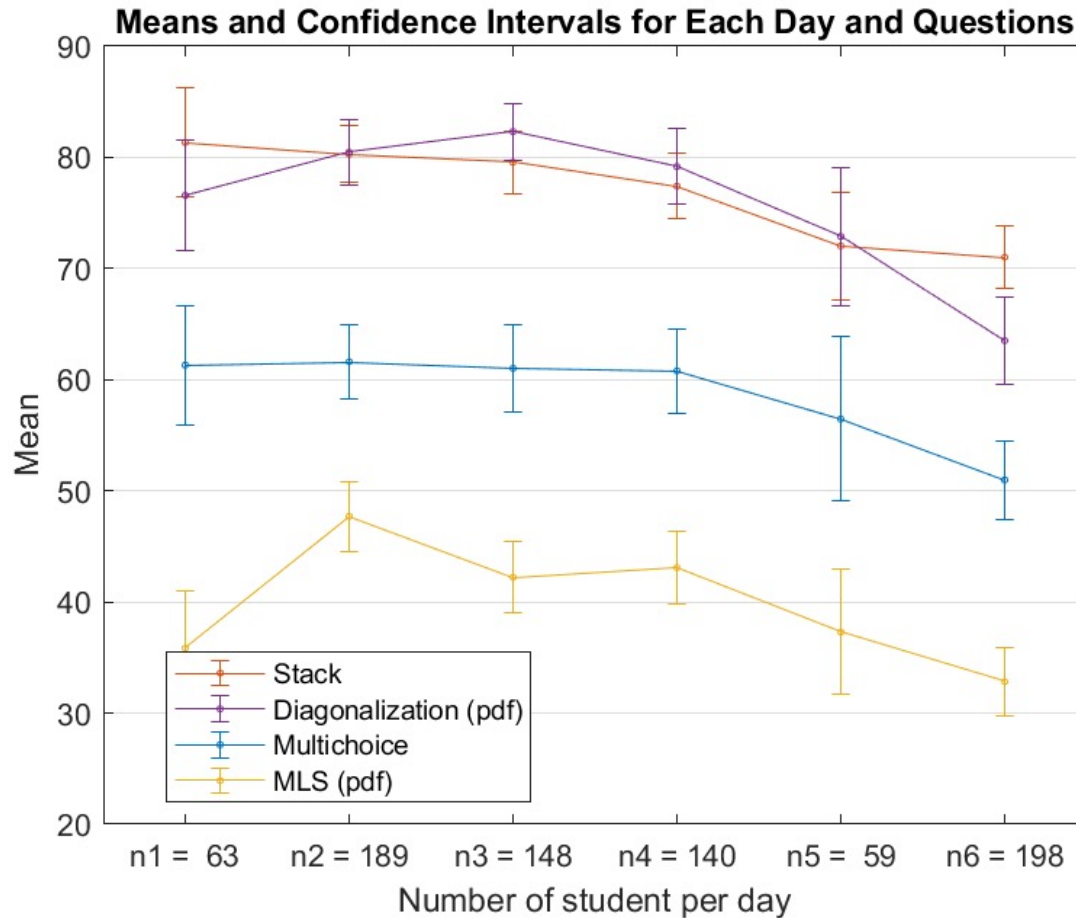
- One small group (5-8 students) and teacher
- Discussion on the week's topics
- Group solves a few harder problems together

Vectors and matrices: final exam

- The final exam was open book remote Moodle exam
- In the exam there were STACK questions, multichoice questions and two questions which required PDF attachment
- The exam was open for one week but the time limit was 3 hours
- Before Covid-19 we had closed book exams in computer labs where we also used STACK questions and students had MATLAB available

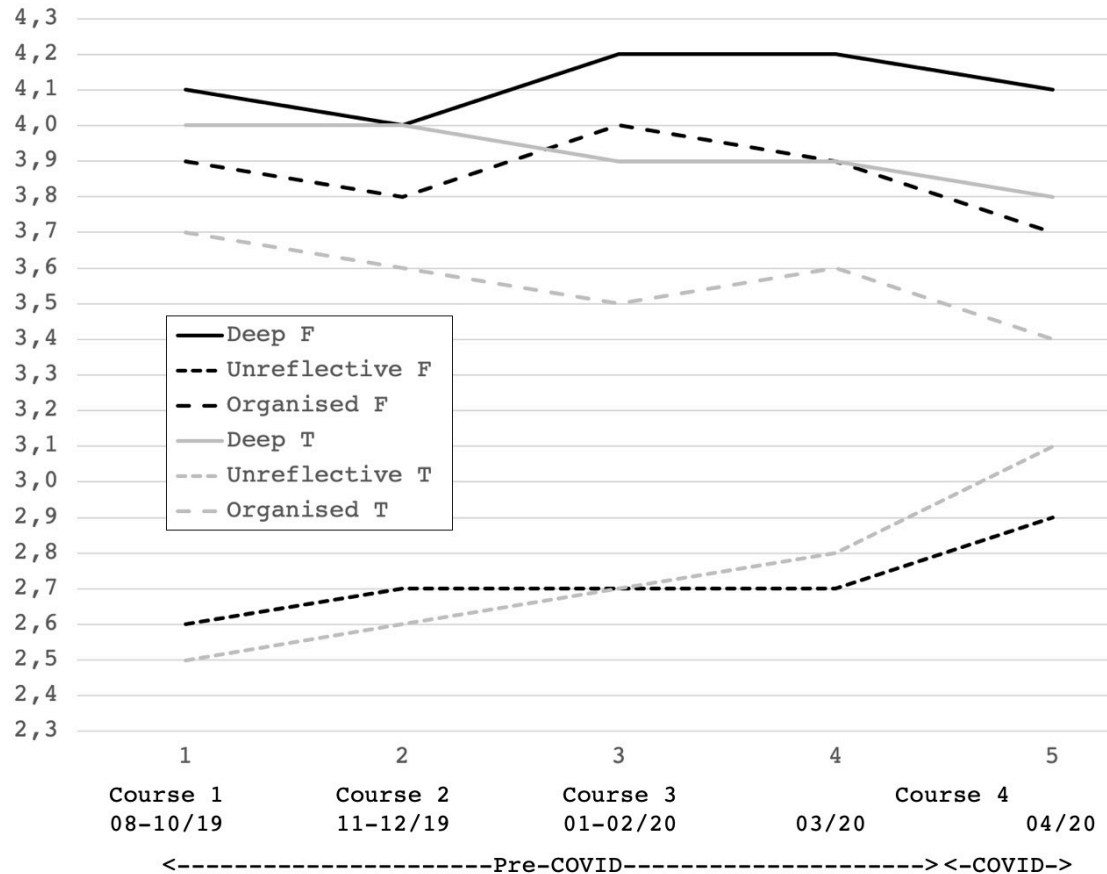
Mean points (0-100) for all four questions: Multichoice, Stack, MLS and Diagonalization

- For Minimum Least Squares and Diagonalization questions students wrote solutions in pdf format with Matlab's Live Editor
- On the last day, all points dropped except for the Stack questions



Some effects of flipping

Benefits from flipping: SAL

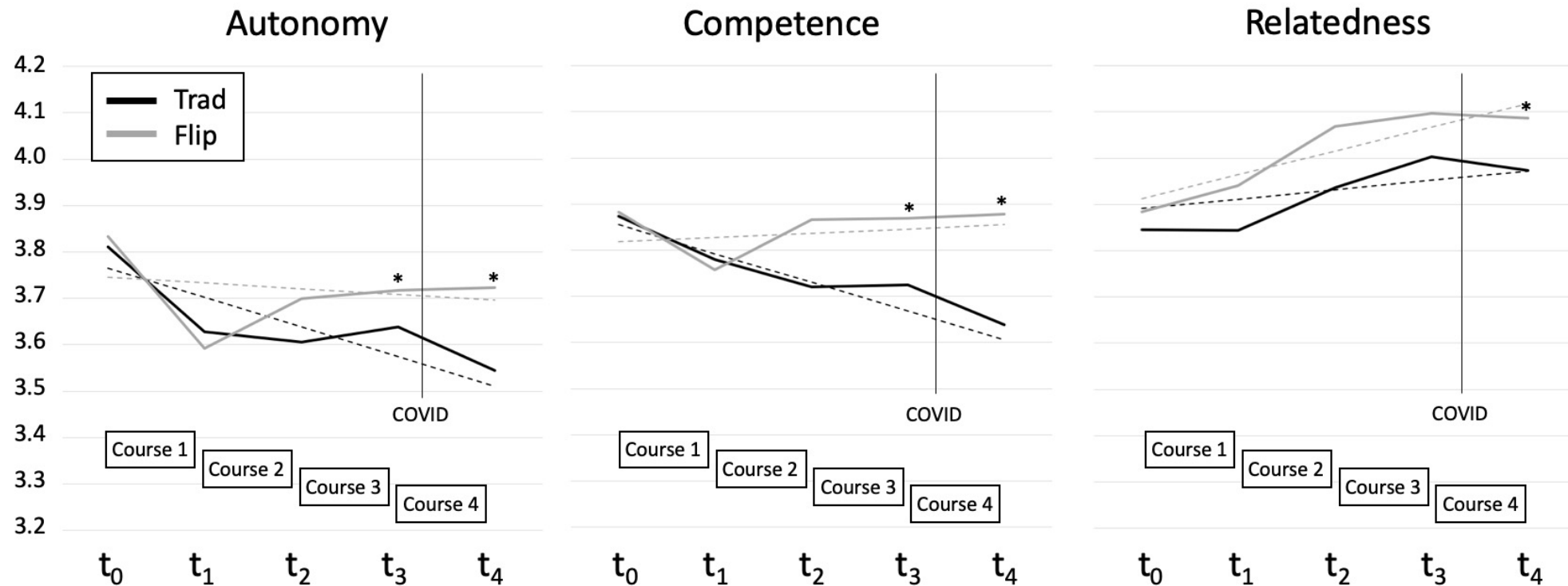


Changes in students' approaches to learning mathematics during first year engineering mathematics courses are more positive on flipped courses:

- Before pandemic, the values for deep and organized approach increase, whereas in the non-flipped model those values decrease.
- After pandemic, students on flipped courses maintain higher values for deep and organized approach.

Benefits from flipping: BPNS

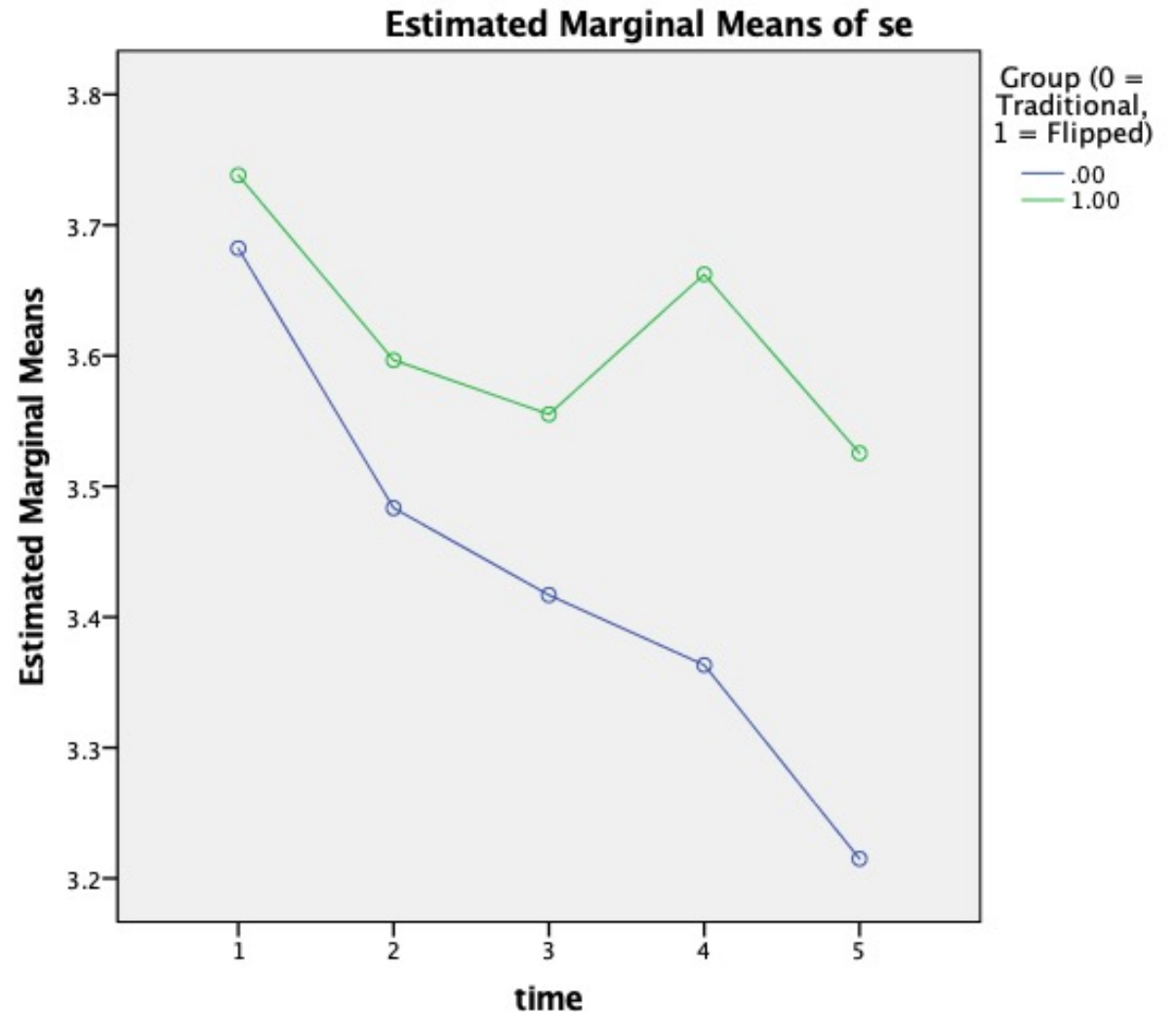
Students' satisfaction of basic psychological needs (autonomy, competence and relatedness) is higher on flipped courses.



Benefits from flipping: SE

On flipped courses, the students' self efficacy starts to increase after an initial drop.

Pandemic causes (again) a dramatic effect.



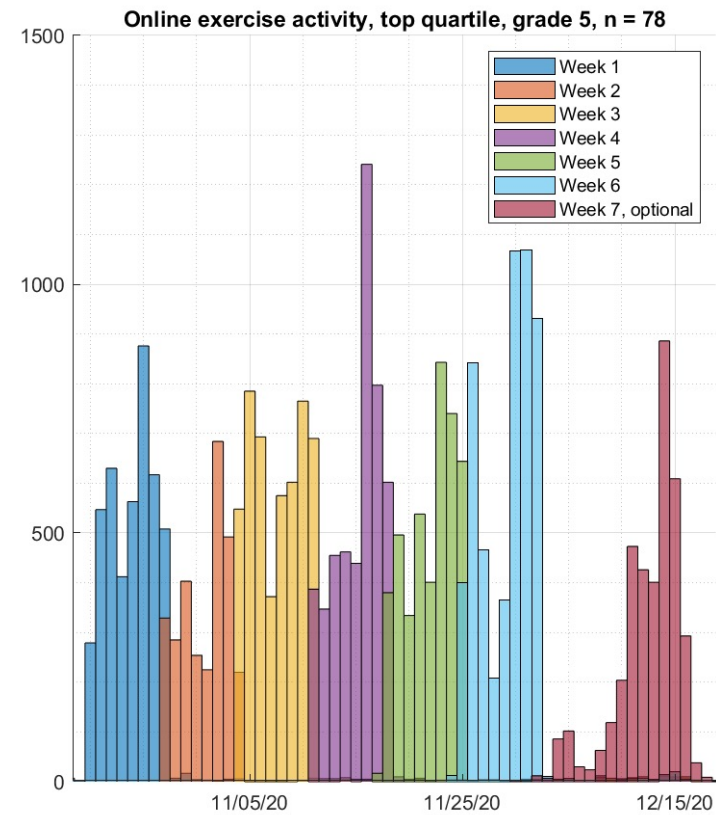
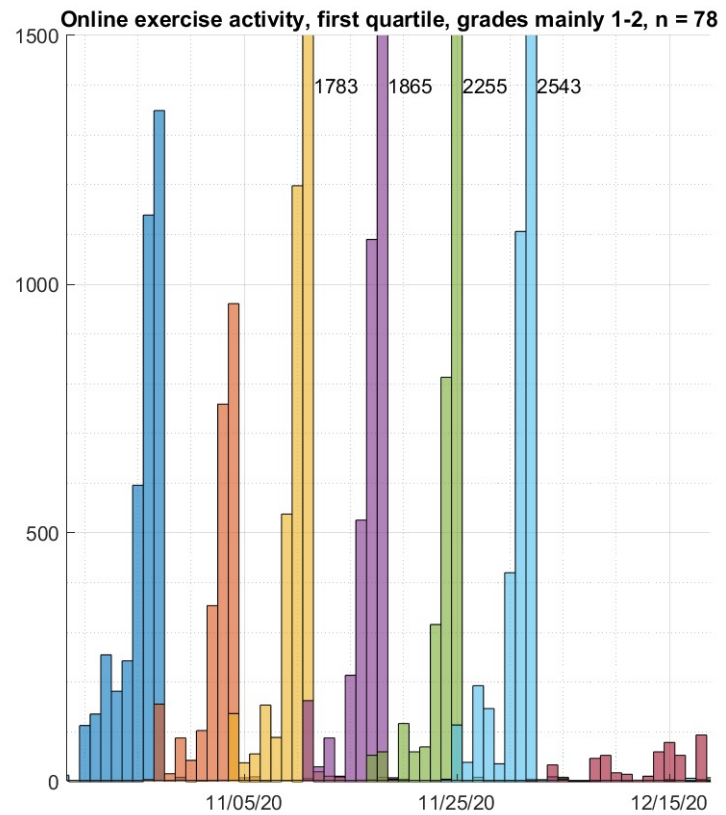
Summary

Flipping math courses has a lot of positive effects.

Flipping mass courses would be very difficult without STACK.

- Carefully planned set of STACK problems on each topic help the students in self-study phase to understand the concepts.
- After working on STACK problems, the students are more able to discuss the topics in group study phase.
- Randomized STACK problems also provide a way to assess the students' learning in remote exams.

Thanks!



References

J. Rämö, P. Nokelainen, E. Viro, T. Kaarakka, R. Kangaslampi, M. Nieminen, J. Hirvonen, and S. Ali-Löytty. Engineering higher education students' approaches to learning in traditional and flipped mathematics courses before and during the COVID-19 pandemic. Submitted, 2020.

P. Nokelainen, I. Puhakka, V. Vuorenpää, J. Rämö, R. Kangaslampi, T. Kaarakka, E. Viro, J. Hirvonen, and S. Ali-Löytty. Longitudinal study of the development of higher education students' basic psychological needs satisfaction during traditional and flipped learning engineering mathematics courses. Submitted, 2020.